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FILTRATION CONVECTION IN A HIGH-FREQUENCY VIBRATION FIELD*

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The effect of high-frequency translational vibrations on the occurrence of filtration convection in a plane horizontal layer of a viscous incompressible liquid saturating a porous medium is studied. Constant temperature is maintained at the boundaries of the layer. It is established that for any vibration direction different from the vertical (transverse) direction, convection in gravity and thermal gravitational convection under both heating from above and heating from below can arise. In the case of reduced gravity, values of the vibration parameter that lead to transition to zero gravity are established. The results are obtained from an analysis of the averaged equations of filtration convection, derived for an arbitrary region.

Convection in a porous medium under nonisothermal conditions occurs widely in nature and engineering and is of interest, for example, for studies of the operation of thermal insulators made of porous materials, which are used in modern equipment both under earth conditions and in space studies. In this connection, it makes sense to study the effect of various factors on filtration convection from the viewpoint of controlling the stability of convective flows in a porous medium. One of these factors is the vibration effect.

Zen'kovskaya [1] studied the effect of high-frequency vertical vibrations on the occurrence of filtration convection in a plane horizontal layer of a liquid saturating a porous medium. It is established that vertical vibrations prevent the occurrence of filtration convection and can even completely suppress it.

The present work is a continuation of [1], where the influence of the vibration direction and velocity on the convective instability in a layer of a viscous incompressible liquid saturating a porous medium at a given transverse temperature gradient is studied. The present investigation is based on an analysis of the averaged equations that are a generalization of the equations obtained in [1]. These equations are derived for an arbitrary region and analyzed in the particular case of a horizontal layer. It is established that for any vibration direction different from the vertical one, destabilizing effects are possible: filtration convection under conditions of zero gravity and occurrence of convective regimes under heating from above in gravity.

Critical values of the thermal and vibration Rayleigh–Darcy numbers are calculated for different directions and velocity of vibration in zero gravity and gravity. In the case of reduced gravity, values for the vibration parameter are obtained for which transition to zero gravity occurs. These results can be used in modeling zero gravity under earth conditions and in controlling convection in a porous medium.

1. Formulation of the Problem and Derivation of the Averaged Equations. Let a vessel D containing a porous medium saturated with a viscous incompressible liquid perform plane simple harmonic oscillations in a specified direction $s = (\cos\varphi, 0, \sin\varphi)$ by the law $a/\omega \cos(\omega t)$. The equations of thermal convection in a porous medium in the Darcy–Oberbeck–Boussinesq approximation, written in a moving coordinate system, have the form [1, 2]

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$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{u}}{\partial t}+\frac{1}{\varepsilon^2}(\boldsymbol{u},\nabla)\boldsymbol{u}=-\frac{\nabla p}{\rho}+g\beta T\boldsymbol{\gamma}-\frac{\nu}{K}\boldsymbol{u}+\boldsymbol{w}_e\beta T,\qquad \operatorname{div}\boldsymbol{u}=0,$$
$$b\frac{\partial T}{\partial t}+(\boldsymbol{u},\nabla T)=\chi\nabla T,\qquad \boldsymbol{w}_e=-\frac{a}{\varepsilon}\omega\cos(\omega t)\boldsymbol{s}. \tag{1.1}$$

Here $\boldsymbol{u}$ is the relative velocity of filtration, T is the temperature reckoned from a certain average value, p is the pressure, ρ is the density, g is the acceleration of gravity, β is the thermal-expansion coefficient, ν is the kinematic viscosity, K is the permeability, ε is the porosity, $b=(\rho c_p)_m/(\rho c_p)_f$ is the ratio of the heat capacities of the medium and the liquid, χ is the effective thermal conductivity, γ is a unit vector directed vertically upward, ω is the oscillation frequency of the vessel, a is the velocity, φ is the angle between the vibration direction and the horizontal axis, and $\boldsymbol{w}_e$ is the translation acceleration. The region $D\subset\mathbb{R}^3$ is considered bounded. However, if periodicity along the space variables x_1 and x_2 is assumed, unbounded regions (layer and cylinder) can also be considered.

The boundary conditions on the solid boundary ∂D are written as

$$u_n\Big|_{\partial D}=0,\qquad \frac{\partial T}{\partial n}+b_T T\Big|_{\partial D}=f, \tag{1.2}$$

which corresponds to the impermeability of the boundary and the general conditions of heat exchange [2]. Here b_T and f are given functions of a points on the boundary and the function b_T can be piecewise continuous, so that on different portions of the boundary it is possible to specify different conditions. The case $b_T\to\infty$ corresponds to specified temperature, and the case $b_T=0$ corresponds to specified heat flux.

We consider oscillation of high frequency ($\omega\to\infty$) with a small amplitude a/ω, assuming that the modulation velocity a remains finite. For system (1.1), (1.2) we employ the known Krylov–Bogolyubov averaging method [3] in the same manner as is done in [4] for a homogeneous liquid and in [1] for a porous medium. According to this asymptotic method, at high frequencies, the unknowns are represented as the sum of fast and slow components. The fast component can be expressed in terms of the slow component, and for the latter, averaging over the fast time leads to a closed system of equations which include a new force called vibrogenic in [5]. This method is known to be effective in studies of mechanical systems in a field of rapidly oscillating forces. A vivid example is the stabilization of the upper position of a pendulum during vertical vibration of the point of suspension. In [1, 4], it is found that vertical vibration of a vessel with a liquid heated from below also stabilizes the relative equilibrium of the liquid. This method gave rise to a new trend in the theory of convection stability — vibration convection — which, beginning with [4], has been the subject of a number of studies (see, for example, [6]).

The averaging method for a homogeneous liquid is substantiated in [7, 8], where it is proved that the stability of a periodic solution of the original problem can be studied by analyzing the stability of a steady solution that is the average over the time interval $2\pi/\omega$ and satisfies the averaged equations. The averaging method for mechanical systems with vibration forces and connections is developed in [5], where a unified treatment of many vibration effects is given.

To derive the averaged equations of filtration convection we write the solution $(\boldsymbol{u},T,p)$ of problem (1.1), (1.2) in the form

$$\boldsymbol{u}=\boldsymbol{v}+\boldsymbol{\xi},\qquad T=\tau+\eta,\qquad p=q+\delta, \tag{1.3}$$

where $\boldsymbol{v}$, τ, and q are slow components and $\boldsymbol{\xi}$, η, and δ are fast components having a zero average over time.

The equations for the fast unknowns can be obtained if we substitute expressions (1.3) into (1.1) and equate vibration terms that are principal in ω: of order ω in the first equation of (1.1) and of order 1 in the second. It should be noted that under ordinary conditions, the permeability coefficient K is rather small, and hence, we assume below that $\nu/K=\lambda\omega$ and λ remains constant as $\omega\to\infty$. Then the system for the fast unknowns takes the form

$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{\xi}}{\partial t}=-\frac{\nabla\delta}{\rho}-\lambda\omega\boldsymbol{\xi}-\frac{a}{\varepsilon}\omega\cos(\omega t)\beta\tau\boldsymbol{s},\qquad \operatorname{div}\boldsymbol{\xi}=0,\qquad b\frac{\partial\eta}{\partial t}=-(\boldsymbol{\xi},\nabla\tau). \tag{1.4}$$

These equations differ from the corresponding equations in [1] by the term $\lambda\omega\boldsymbol{\xi}$ in the first equation, which should be taken into account if $\lambda = O(1)$ as $\omega \to \infty$. As shown below, this leads to redefinition of the Grashof–Darcy vibration number. Eliminating the pressure δ from (1.4) by using the projector Π [9], for the unknowns $\boldsymbol{\xi}$ and η, we obtain the system

$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{\xi}}{\partial t} = -\lambda\omega\boldsymbol{\xi} - \frac{a}{\varepsilon}\omega\beta\cos(\omega t)\boldsymbol{w}, \qquad \operatorname{div}\boldsymbol{\xi} = 0, \qquad b\frac{\partial \eta}{\partial t} = -(\boldsymbol{\xi}, \nabla\tau). \tag{1.5}$$

Here $\boldsymbol{w} = \Pi(\boldsymbol{s}\tau)$, where Π is an orthoprojector in $L_2(D)$ for a subspace of solenoidal vectors with a normal component w_n equal to zero at the boundary ($\boldsymbol{n}$ is an outer normal unit vector). This means that the vector $\boldsymbol{w}$ can be written as $\boldsymbol{w} = \boldsymbol{s}\tau - \nabla\Phi$, where the function Φ is a solution of the Neumann problem

$$\Delta\Phi = \operatorname{div}\boldsymbol{s}\tau, \qquad \frac{\partial\Phi}{\partial n}\Big|_{\partial D} = (\boldsymbol{s}\tau, n). \tag{1.6}$$

Integrating system (1.5) with respect to the explicitly included time and considering the temperature τ and the vector $\boldsymbol{w}$ constant on the segment $[0, 2\pi/\omega]$, we obtain

$$\boldsymbol{\xi} = (B\sin(\omega t) + C\cos(\omega t))\boldsymbol{w}, \qquad \eta = \frac{1}{\omega b}(B\cos(\omega t) - C\sin(\omega t))(\boldsymbol{w}, \nabla\tau),$$
$$B = \frac{-a\beta}{1 + \lambda^2\varepsilon^2}, \qquad C = \lambda\varepsilon B, \qquad \lambda = \frac{\nu}{K\omega}. \tag{1.7}$$

Substituting (1.7) and (1.3) into (1.1), (1.2) and averaging over the explicitly contained time, for smooth components $\boldsymbol{v}$, τ and q, we obtain a closed autonomous system

$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{\varepsilon^2}(\boldsymbol{v}, \nabla)\boldsymbol{v} = -\frac{\nabla q}{\rho} + g\beta\tau\boldsymbol{\gamma} - \frac{\nu}{K}\boldsymbol{v} + \frac{a^2\beta^2}{2\varepsilon^2(1 + \lambda^2\varepsilon^2)}(\boldsymbol{w}, \nabla)\Big(\frac{\varepsilon}{b}\boldsymbol{s}\tau - \boldsymbol{w}\Big),$$
$$\operatorname{div}\boldsymbol{v} = 0, \qquad b\frac{\partial\tau}{\partial t} + (\boldsymbol{v}, \nabla\tau) = \chi\Delta\tau, \qquad \boldsymbol{w} = \boldsymbol{s}\tau - \nabla\Phi, \qquad \operatorname{div}\boldsymbol{w} = 0, \tag{1.8}$$
$$v_n\Big|_{\partial D} = w_n\Big|_{\partial D} = 0, \qquad \frac{\partial\tau}{\partial n} + b_T\tau\Big|_{\partial D} = f.$$

In problem (1.8), we convert to dimensionless variables by choosing the scales as follows: $(x, t, \boldsymbol{v}, \tau, q) \to (l, l^2/\nu, \nu/l, Al, \rho\nu^2/K)$, where l is the characteristic dimension of the cavity and A is the characteristic value of the temperature gradient. Denoting the dimensionless variables by the same characters as the dimensional ones, we obtain the following system of equations and boundary conditions:

$$c\frac{\partial \boldsymbol{v}}{\partial t} + \frac{c}{\varepsilon}(\boldsymbol{v}, \nabla)\boldsymbol{v} = -\nabla q + \mathrm{Gr}\boldsymbol{\gamma}\tau - \boldsymbol{v} + \mathrm{Gv}(\boldsymbol{w}, \nabla)\Big(\frac{\varepsilon}{b}\boldsymbol{s}\tau - \boldsymbol{w}\Big),$$
$$\operatorname{div}\boldsymbol{v} = 0, \qquad b\frac{\partial\tau}{\partial t} + (\boldsymbol{v}, \nabla\tau) = \frac{1}{\mathrm{Pr}}\Delta\tau, \qquad \boldsymbol{w} = \boldsymbol{s}\tau - \nabla\Phi, \qquad \operatorname{div}\boldsymbol{w} = 0; \tag{1.9}$$

$$u_n\Big|_{\partial D} = w_n\Big|_{\partial D} = 0, \qquad \frac{\partial\tau}{\partial n} + b_T l\tau\Big|_{\partial D} = \frac{f}{A}. \tag{1.10}$$

Here $c = K/(l^2\varepsilon)$ is the ratio of the dimensionless permeability to the porosity, $\mathrm{Pr} = \nu/\chi$ is the Prandtl number, $\mathrm{Gr} = A\beta g l^2 K/\nu^2$ is the Grashof–Darcy filtration number, and $\mathrm{Gv} = (a\beta Al/\nu\varepsilon)^2 K/(2(1 + \lambda^2\varepsilon^2))$ is the Grashof vibration number for the porous medium. The averaged system obtained in [1] corresponds to $\lambda = 0$, i.e., it differs by the definition of the Grashof vibration number and by the expressions for the fast components.

2. Mechanical Equilibrium and Its Stability. Necessary (and sufficient for a simply connected region) conditions for the existence of an equilibrium solution ($\boldsymbol{v}_0 = 0$) are given in [1]. It follows from these conditions that equilibrium is possible only under special heating conditions and special geometry of the region D. The liquid can be in equilibrium only when the temperature distribution is linear along the vertical coordinate. This is the case, for example, in a horizontal layer or an infinite circular cylinder. For a rectangle, the thermal-insulation condition $\partial T/\partial n = 0$ can be specified on the vertical walls. Below, we consider the case where the region D is a plane layer $|y| \leqslant l/2$. On the solid boundaries of this region, temperatures T_1 and T_2

are specified so that the characteristic gradient is $A = (T_1 - T_2)/l$, where $A > 0$ for heating from below and $A < 0$ for heating from above. Problem (1.9), (1.10), (1.6) has a steady solution

$$\begin{gathered} v_0 = 0, \qquad \tau_0 = -y, \qquad q_0 = -\mathrm{Gr}\frac{y^2}{2} + \mathrm{const}, \qquad w_{0x} = -y\cos\varphi, \\ w_{0y} = 0, \qquad \Phi_0 = -\frac{y^2}{2}\sin\varphi + \mathrm{const}. \end{gathered} \tag{2.1}$$

We note that for this solution, the velocity, temperature, and pressure do not depend on the vibration parameters, and, hence, the problem of filtration convection with no vibration forces has the same equilibrium solution. We study the stability of solution (2.1) against small plane perturbations of $\boldsymbol{u}, T, \boldsymbol{w}$, and Φ.

We introduce the stream function by the relations

$$u_x = \frac{\partial\psi}{\partial y}, \qquad u_y = -\frac{\partial\psi}{\partial x}, \qquad w_x = \frac{\partial F}{\partial y}, \qquad w_y = -\frac{\partial F}{\partial x}.$$

Then, the linearized system has the form

$$\begin{gathered} -c\mathrm{Pr}\frac{\partial^2\Delta\psi}{\partial t\partial x} = R\frac{\partial^2 T}{\partial x^2} + \frac{\partial\Delta\psi}{\partial x} + \mu\Big[\cos^2\varphi\frac{\partial^2 T}{\partial x^2} + \sin\varphi\frac{\partial^3 F}{\partial x^3} - \cos\varphi\frac{\partial^3 F}{\partial x^2\partial y} - \Big(1 - \frac{b}{\varepsilon}\Big)w_{0x}\frac{\partial^2\Delta F}{\partial x^2}\Big], \\ w_{0x} = -y\cos\varphi, \qquad \mathrm{Pr}b\frac{\partial T}{\partial t} = \Delta T - \frac{\partial\psi}{\partial x}\mathrm{Pr}, \qquad \Delta F = \frac{\partial T}{\partial y}\cos\varphi - \frac{\partial T}{\partial x}\sin\varphi, \\ \psi = F = T = 0 \quad \text{for} \quad y = \pm 1/2. \end{gathered} \tag{2.2}$$

Here $R = \mathrm{Pr}\cdot\mathrm{Gr}$ and $\mu = \mathrm{Gv}\cdot\mathrm{Pr}\varepsilon/b$ are the Rayleigh–Darcy thermal and vibration numbers. Following [2], we set the parameter c equal to zero. Indeed, the permeability coefficient K is on the order of 10^{-12}–10^{-8} cm^2, and even for very porous fibrous metals, the value of K does not exceed 10^{-4} cm^2. The porosity varies in the range $0 < \varepsilon < 1$; for metals, it is close to unity, and for fills of close-packed spheres, $0.25 < \varepsilon < 0.5$. Hence, the coefficient c can be set equal to zero, as, for example, when the Darcy model is used to describe filtration convection.

From (2.2,) considering normal perturbations of the form

$$\Big(\frac{\partial\psi}{\partial x}(x,y,t), T(x,y,t), F(x,y,t)\Big) = \exp(\sigma t + i\alpha x)(\mathrm{Pr}^{-1}u(y), \Theta(y), f(y)),$$

we obtain the spectral problem

$$\begin{gathered} Lu = R\alpha^2\Theta + \mu\alpha^2\Big[\cos^2\varphi\Theta + i\alpha\sin\varphi f - \cos\varphi Df - \Big(1 - \frac{b}{\varepsilon}\Big)w_{0x}Lf\Big], \qquad w_{0x} = -y\cos\varphi, \\ \mathrm{Pr}b\sigma\Theta = L\Theta - u, \qquad Lf = \cos\varphi D\Theta - i\alpha\sin\varphi\Theta, \qquad u = \Theta = f = 0 \quad \text{for} \quad y = \pm 1/2. \end{gathered} \tag{2.3}$$

Here $D \equiv d/dy$ and $L \equiv D^2 - \alpha^2$ (α is the wave number).

System (2.3) has constant coefficients for $b = \varepsilon$ and $\varphi = \pi/2$. In [1], it is proved that in these cases the monotonicity principle holds (instability is caused by monotonic perturbations). In [10], where the case $b = \varepsilon$ is considered, the solution of problem (2.3) was reduced to studying a transcendental equation whose left side was a sixth-order determinant. In the present paper, we assume that the ratio b/ε is arbitrary, $0 \leqslant \varphi \leqslant \pi/2$, $\mathrm{Re}\,\sigma = 0$, and $\mathrm{Im}\,\sigma \neq 0$. To solve the spectral problem (2.3), we employ a shooting method which reduces the boundary-value problem to the Cauchy problem, and in eigenvalue problems, to a numerical solution of a complex transcendental equation having the form of a determinant. The Cauchy problems were solved by the Runge–Kutta–Fellberg method of the fifth order using the RKF45 subprogram [11] as one spectral parameter The frequency of neutral oscillations $\mathrm{Im}\,\sigma$ was used as one spectral parameter, and the other spectral parameter was the vibration Rayleigh–Darcy number $\mu(\alpha, \varphi)$ for $R = 0$ (convection in zero gravity) or the thermal filtration Rayleigh number $R(\alpha, \varphi, \mu)$ (gravitational convection). We note that the calculations performed did not reveal vibration instability (on the stability boundary, the imaginary term σ is small: $\mathrm{Im}\,\sigma \sim 10^{-8}$), which numerically confirms the monotonicity principle.

TABLE 1

φ, deg	$b/\varepsilon = 0.5$		$b/\varepsilon = 0.8$		$b/\varepsilon = 1$		$b/\varepsilon = 1.25$	
	μ_*	α_*	μ_*	α_*	μ_*	α_*	μ_*	α_*
0	66.79	3.25	52.63	3.25	46.42	3.26	40.56	3.24
30	114.67	2.88	85.65	2.99	73.87	2.99	63.39	2.92
45	274.35	2.17	193.38	2.06	157.77	2.50	130.79	2.45
50	421.64	1.87	284.64	2.04	233.37	2.15	190.59	2.19
52	512.07	1.74	342.99	1.93	279.74	2.01	227.12	2.06
54	630.45	1.62	419.34	1.78	340.43	1.87	274.84	1.92
56	787.54	1.50	521.00	1.62	420.94	1.73	338.05	1.78
58	999.23	1.39	657.28	1.52	529.39	1.58	423.08	1.65
60	1289.60	1.27	844.53	1.40	677.95	1.46	539.52	1.50
65	2664.77	1.01	1730.44	1.11	1380.03	1.16	1088.55	1.51
70	6504.85	0.78	4199.75	0.85	3333.80	0.89	2664.57	0.94

TABLE 2

φ, deg	r	μ_*	α_*	R	φ, deg	r	μ_*	α_*	R
	1	39.10	3.239	6.253		1	487.14	1.560	22.071
	5	44.85	3.255	1.339		5	636.77	1.481	5.047
0	10	45.63	3.257	0.675	60	10	657.16	1.472	2.564
	∞	46.42	3.259	0		∞	677.95	1.464	0
	1	120.10	2.531	10.959		1	2368.9	0.950	48.671
	5	149.51	2.466	2.445		5	3126.9	0.901	11.184
45	10	153.57	2.459	1.239	70	10	3229.5	0.896	5.683
	∞	157.77	2.498	0		∞	3333.8	0.891	0

3. Convection in Zero Gravity. It is known that for $g = 0$ there is no thermal convection. One effect due to high-frequency vibration is the possibility of convection occurring in zero gravity. For a homogeneous liquid, this problem was studied in [12–14]. As shown in [1], with transverse vibration ($\varphi = \pi/2$), filtration convection cannot arise in zero gravity. Below, it is established analytically and numerically that in any vibration direction containing a longitudinal component, convection can arise in a plane layer when $g = 0$.

For $\varphi = 0$, $R = 0$, and $\alpha \neq 0$, problem (2.3) has only positive and simple eigenvalues $\mu(\alpha)$, and for $\varphi = \pi/2$, all eigenvalues $\mu(\alpha) < 0$. These statements can be rigorously proved mathematically on the basis of the theory of oscillation operators of M. G. Krein and F. R. Gantmakher, in the same manner as is done in the work of V. I. Yudovich, who employed this theory for problems of hydrodynamic stability [15]. For $0 < \varphi < \pi/2$, the existence of positive eigenvalues $\mu(\alpha)$ is established numerically.

Assuming that $R = 0$ in (2.3), we obtain numerical critical values $\mu_*(\varphi, b/\varepsilon) = \min_\alpha \mu(\alpha, \varphi, b/\varepsilon)$. The parameter b/ε take values from 0.5 to 2, which are typical of the most widely occurring porous media. Filtration convection in porous layers of different orientation was studied in [16], where calculation results for the main characteristics of heat transfer are given and compared with experimental data. The parameter values are taken from the tables of [16]. Figure 1 (for $b/\varepsilon = 1$) and Table 1 (for $b/\varepsilon = 0.5$, 0.8, 1, and 1.25) show dependences $\mu_*(\varphi)$ [and $\alpha_*(\varphi)$ in Table 1], from which it follows that the vibration Rayleigh–Darcy number

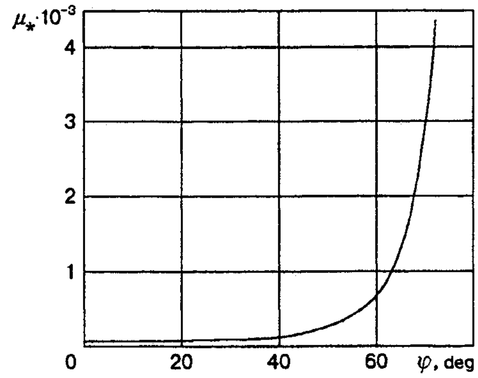

Fig. 1

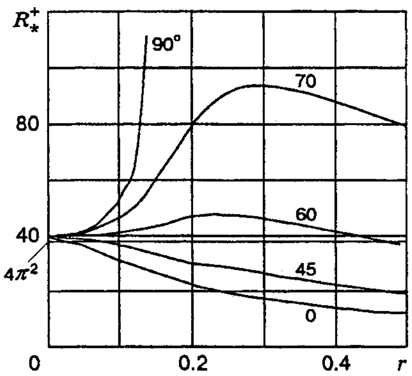

Fig. 2

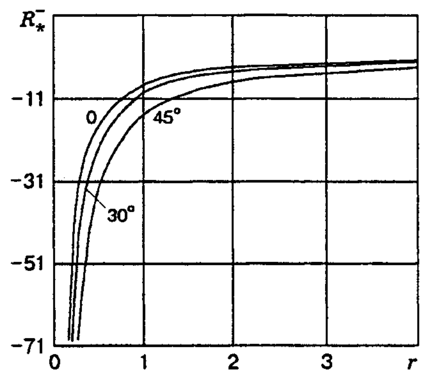

Fig. 3

$\mu_*(\varphi)$ increases with increase in the angle φ, so that the stability region is extended. For fixed angle φ, the stability region decreases as the ratio b/ε increases. For $b/\varepsilon = 1$ and $\varphi \neq \pi/2$, the asymptotic behavior of the vibration Rayleigh number $\mu(\alpha)$ as the wave numbers $\alpha \to 0$ is given by

$$\mu(\alpha) = \frac{120}{\alpha^2 \cos^2 \varphi} + O\Big(\frac{1}{\alpha}\Big). \tag{3.1}$$

Beginning with $\alpha = 0.1$ the numerical and asymptotic values calculated from formula (3.1) coincide with an accuracy of 1%.

4. Gravitational Convection. If $g \neq 0$, the vibration Rayleigh number μ can be written as $\mu = r^2 R^2$, where the parameter $r^2 = (a/(\sqrt{2}gl))^2 \chi\nu/(Kb\varepsilon(1+\lambda^2\varepsilon^2))$ does not depend on temperature and characterizes the ratio of the vibration and gravitation forces. If $\varphi = \pi/2$ there are only positive Rayleigh numbers $R(\alpha, r)$ [1]. A numerical study of problem (2.3) shows that for $\varphi \neq \pi/2$ there are both positive $R^+(\alpha, r, \varphi)$ and negative $R^-(\alpha, r, \varphi)$ values of the Rayleigh numbers. The positive values correspond to heating from below, and the negative values corresponds to heating from above. It is known that in the absence of vibration, convection is possible only for $R > 0$.

Figure 2 shows neutral curves $R_*^+(r, \varphi) = \min\limits_\alpha R^+(\alpha, r, \varphi)$ for $b/\varepsilon = 1$ and $\varphi = 0$, 45, 60, 70, and 90°. The curve $R_*(r) = 4\pi^2$ corresponds to the value of the filtration Rayleigh number in the absence of vibration. It is evident from the plots that the vertical direction is unique — only in this case is absolute stabilization possible. For angles $55° \leqslant \varphi < 90°$, there are values of the parameter r for which maximum stability takes place, and for $0° \leqslant \varphi < 55°$ vibration has only a destabilizing effect. Figure 3 shows neutral curves $R_*^-(r, \varphi) = -\min\limits_\alpha |R^-(\alpha, r, \varphi)|$ for $\varphi = 0, 30, 45°$ and $b/\varepsilon = 1$, which exist only for $r \neq 0$. The behavior of the critical values of the parameters $\mu_*(r, \varphi, R)$ and $\alpha_*(r, \varphi, R)$ for $r \geqslant 1$ is reflected in Table 2. For rather large r ($r > 10$), the critical values of the parameters μ and α approach practically the same values as in the case of zero gravity ($r = \infty$). This fact can be used in modeling zero gravity under earth conditions in studies of filtration convection in a high-frequency vibration field.

Conclusion. According to the studies performed, as in the case of Oberbeck–Boussinesq convection, high-frequency vibrations can have both stabilizing and destabilizing effects on the convective instability of a liquid saturating a porous medium. It is possible to retard convection or even completely prevent it by making the vessel perform vertical vibrations. At the same time, intense horizontal vibration has a destabilizing effect, giving rise to convection in zero gravity and microgravity. Under earth conditions, horizontal vibration causes convection with both heating from below and heating from above.

Thus, vibration of the vessel can be used to control convective instability in the porous medium.

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